Features of OFDM signals delay tracking for navigation and radio location

I. V. Korogodin

National Research University Moscow Power Engineering Institute, Russia

Abstract— The orthogonal frequency multiplexing as a modulation technique for measuring signals of navigation and location systems is discussed. It is shown that the technique has several advantages in comparison on traditional BOC and BPSK. An OFDM with irregular subcarriers approach is proposed. Relatively to OFDM with uniform subcarriers the approach allows to increase the delay estimation accuracy. An anti-jam capability is increased proportionally to the number of beacons. It is possible to generate the signal with the constant envelope and to process signal components separately.

1. INTRODUCTION

The article discusses advantages, disadvantages and features of OFDM (orthogonal frequency-division multiplexing) signals in the context of delay estimation in radio location and navigation systems. Any OFDM signal is the sum of different signals, each signal uses own subcarrier. There are from tens to thousands subcarriers in modern OFDM signals.

This kind of modulation is wide spreaded in telecommunication systems like WiFi, dedicated short-range communications (DSRC), digital television (DVB) and others. The reason of the prevalence is good spectrum utilization and sustainability into multipath propagation.

These advantages of the OFDM signals are in demand for location and navigation purposes. The multipath propagation is the main reason of positioning accuracy degradation in urban conditions for modern navigation systems. Several prototypes and commercial devices using OFDM telecommunication signals for delay and angle-of-arrival estimations are known [1], [2]. But the telecommunication signals are not intended for the measurements.

Has the multiplexing any advantages and features over BOC and BPSK modulation schemes for the navigation and location systems?

The study [3] is devoted to investigations of new navigation S-band signals. It states that OFDM has potential for navigation, but the authors uses existing DVB telecommunication signals for their calculations.

It is actual to research OFDM signals delay estimation performance and to choose the signals parameters to maximize the performance in location and navigation systems.

2. POWER DISTRIBUTION REQUIREMENTS

The delay estimation performance is defined by a signal power distribution into the allocated band.

The delay accuracy depends on several factors:

- thermal noise of the receiver
- multipath conditions
- intentional, unintentional or intrasystemic interference

In addition, power distribution defines other important system properties:

- acquisition performance (in case of limited computing resources)
- transmitter complexity and efficiency
- receiver power consumption

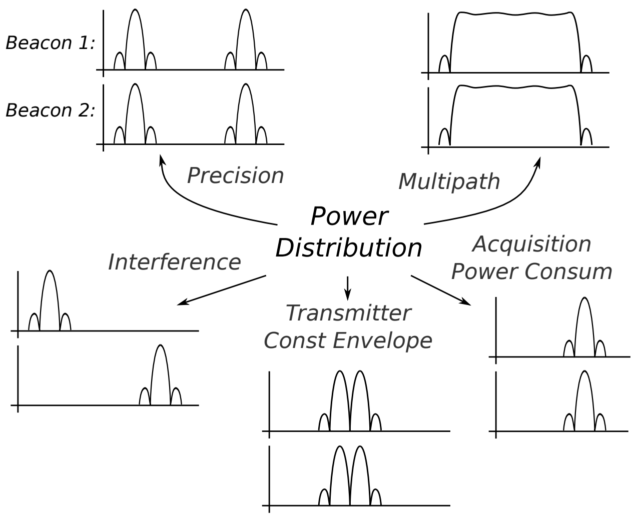

Figure 1: Requirements to the spectrum of the signal

The factors generate contradictory requirements for a signal spectrum shape (fig. 1).

In conformity with Woodward equation a potential dispersion (caused by the **thermal noise** only) for the delay estimation is in an invert proportion with the square of the signal effective bandwidth:

$$\sigma_{\tau,n}^2 \sim \frac{1}{\beta^2}, \tag{1}$$

$$\beta^2 = \int_{-\infty}^{-\infty} (2\pi f)^2 N\left(f\right) df, \tag{2}$$

where $N\left(f\right)$ is a normalized signal power spectrum density.

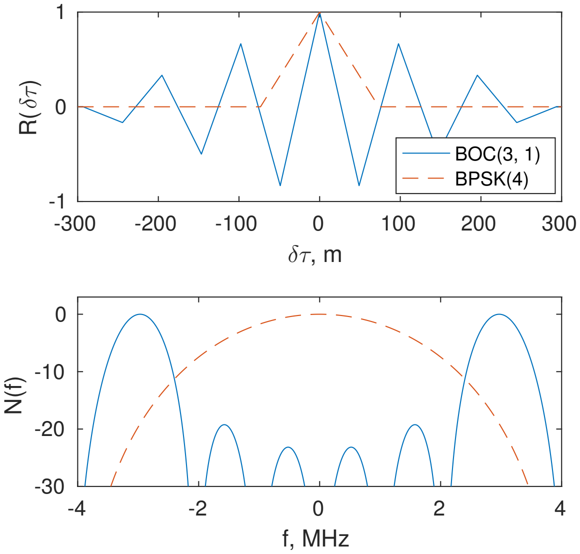

Figure 2: Autocorrelation functions for signals with the equal bandwidth

As conclusion, we should focus the spectrum power on the edges of the allocated band in order to maximize accuracy in good receiving conditions. For example, there are autocorrelation functions and spectrums for $BPSK(4)$ and $BOC(3,1)$ signals on the fig. 2. The signals have the same bandwidth, but the spectrum of the BOC signal is shifted to the edges of the allocated band.

As result, the BOC autocorrelation function has a narrower peak. The $BOC(3,1)$ has the delay potential dispersion 4.5 times smaller than $BPSK(4)$ [5].

The potential accuracy is not achievable in real systems. The main reason of delay accuracy degradation is **multipath** conditions of the signal propagation. For multipath mitigation the spectrum power should be located evenly into the allocated band, and not only on the edges. It can be illustrated in time and frequency domains.

In the time domain, autocorrelation function for the BOC signal (the signal with the spectrum on the edges) has many additional peaks (fig. 2). Reflected signals cause distortions in the delay locking loops through the peaks.

In the frequency domain, we should consider a transmission function of a channel with the multipath conditions (fig. 3). The channel is named *fading channel* because it suppresses harmonics at some frequencies. Direct and reflected signals harmonics interfere, they are added or subtracted in depending on their frequency and delays. Phase and amplitude of resulting sum are different from parameters of the direct signal, it causes errors in delay estimations. Moreover, the harmonics are disturbed more in the hollows of transmission function. A signal with narrowband components tends to fast change the multipath error when its delay varies. The components fall into the hollows, then at the maximums.

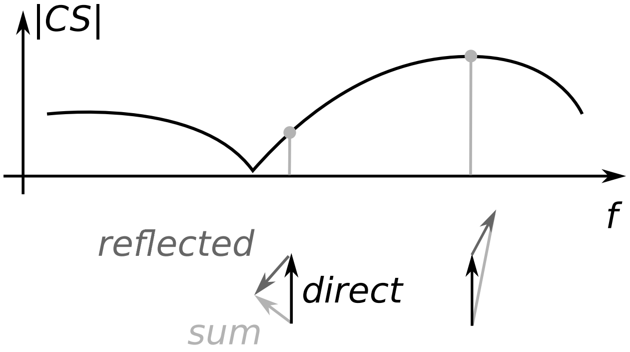

Figure 3: A channel transmission function

Signals from different beacons should be placed on different frequencies for anti-jam capability. The anti-jam capability increasing is in a direct proportion to the amount of the beacons. It decreases the intrasystemic **interference** too. The low level of the intrasystemic interference is a big advantage for a signal acquisition procedure.

The **acquisition** procedure characteristics depend on a complexity of the signal structure. The acquisition core can achieve better performance for narrowband simple signals. The narrow bandwidth allows to decrease a power consumption of a RF receiver part.

The signal structure defines a transmitter architecture. It's desirable to have a constant envelope for the signal to increase efficiency of transmission. The signal should be a sum of several BPSK or BOC signals with equal power. In this case, it is possible to construct a resulting signal with the constant envelope by means of the time domain multiplexing.

3. OFDM SIGNALS FOR DELAY ESTIMATIONS

OFDM allows to form spectrums with a complex shape and to process different subcarries separately. Let's use the possibilities to achieve a compromise on the above requirements.

A signal mathematical model can be depicted as:

$$s\left(t\right)=\sum_{k=1}^{K}s_k\left(t\right)=\sum_{k=1}^{K}\sqrt{2P_k}G_k\left(t-\tau\right)\cos\left(\left(\omega_0+\omega_k\right)t+\varphi\right) \tag{3}$$

where s_k is signal component on the k-th subcarrier, K is a number of the subcarriers, $\omega_0=2\pi f_0$, f_0 is a carrier frequency, $\omega_k=2\pi f_k$, $f_k=\frac{1}{2}\left(2k-K-1\right)\delta f$ is $k-th$ subcarrier frequency, δf is spacing between the subcarriers, G_k is a modulation sequence, P_k is the total power of the k-th signal component.

The telecommunication signals usually have a big number of subcarriers K (up to thousands) with equal power ($P_i=P_j, i,j\in\{1,...,K\}$). As shown above, for navigation signals it is required to increase the proportion of the outermost subcarriers in the total signal power in order to achieve high potential accuracy. At the same time, in order to reduce the influence of multipath, average subcarriers are needed.

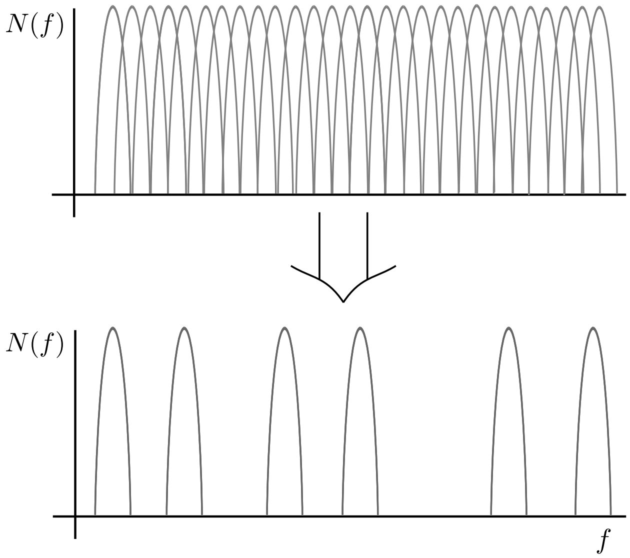

Figure 4: Signal spectrum for the proposed modulation scheme

Obviously, the optimum lies between these two extremes. But we cannot just vary power of certain subcarriers P_k. All used subcarriers must have the same power to save the constant envelope of the signal. In this paper, it is proposed to use **OFDM signals with an irregular set of subcarriers** to achieve a compromise to the specified requirements (fig. 4). The signal contains components on the utmost subcarriers and a small number of components between them, spaced irregularly:

$$s\left(t\right)=\sum_{k\in\mu}\sqrt{2P_k}G_k\left(t-\tau\right)\cos\left(\left(\omega_0+\omega_k\right)t+\varphi\right) \tag{4}$$

where $P_k=P=const$, $\mu=\{1,k_1,k_2,...,K\}$ is a mask of subcarriers numbers, μ has up to 10-20 items.

3.1. Delay accuracy improvement

The power of the signal is redistributed to the extreme subcarriers relative to the traditional OFDM signal with a uniform distribution of subcarriers. It increases the potential **accuracy of delay** estimates in comparison with uniform power distribution:

$$\frac{\sigma^2_{\tau,n,uni}}{\sigma^2_{\tau,n,irreg}}=\frac{\beta^2_{\tau,n,irreg}}{\beta^2_{\tau,n,uni}}\approx\frac{\sum\limits_{k\in\mu}\omega_k^2\frac{K}{N}}{\sum\limits_{k=1}^{K}\omega_k^2} \tag{5}$$

where $N=|\mu|$ is the number of items in μ. For example, for $K=32$, $N=6$ (as it shown on the figures, the numbers are just chosen for an idea illustration) a mean value of the proportion is about $E_\mu\left[\frac{\sigma^2_{\tau,n,uni}}{\sigma^2_{\tau,n,irreg}}\right]\approx1.7$.

There is the multipath propagation in most practical applications and the potential accuracy is unachievable. Let's compare delay estimation Cramer-Rao Bound (CRB) for OFDM signals with uniform and irregular subcarriers. Equations for OFDM delay CRB in the multipath condition are derived in study [4]. The equations are valid for the simple case of a known initial carrier phase, but it is enougth for the demonstration. We can generalize the equations to the case of modulated subcarriers.

We consider delay (ranging) estimation using OFDM signals in multipath channels. The channel impulse response is:

$$h\left(t\right)=\alpha_1\delta\left(t-\tau_1\right)+\alpha_2\delta\left(t-\tau_2\right) \tag{6}$$

where α_1, τ_1 are the direct signal attenuation and delay, α_2, τ_2 are the reflected signal attenuation and delay. They are unknown and form $\theta = [\alpha_1, \tau_1, \alpha_2, \tau_2]^T$.

We receive the direct and reflected signals with additive white Gaussian noise (power spectrum density N_0) during time interval $[0; T]$:

$$y(t) = \int_{-\infty}^{T} h(u)\, s(t-u)\, du + n(t) \tag{7}$$

Then Fisher information matrix (FIM) for θ is:

$$\mathbf{J}_\theta = \int_{f \in Band} \mathbf{J}(2\pi f) df \tag{8}$$

where $\mathbf{J}(2\pi f)$ is the FIM for a one harmonic:

$$\mathbf{J}(\omega) = \begin{vmatrix} \frac{\alpha_1^2 T}{N_0} P_s(\omega)\,\omega^2 & \dots & \dots & \dots \\ 0 & \frac{TP_s(\omega)}{N_0} & \dots & \dots \\ \frac{\alpha_1 \alpha_2 T}{N_0} P_s(\omega)\,\omega^2 \cos(\omega\tau) & \frac{\alpha_2 T}{N_0} P_s(\omega)\,\omega \sin(\omega\tau) & \frac{\alpha_2^2 T}{N_0} P_s(\omega)\,\omega^2 & \dots \\ -\frac{\alpha_1 T}{N_0} P_s(\omega)\,\omega \sin(\omega\tau) & \frac{TP_s(\omega)}{N_0}\omega^2 \cos(\omega\tau) & 0 & \frac{TP_s(\omega)}{N_0} \end{vmatrix} \tag{9}$$

where $\tau = \tau_2 - \tau_1$ is the difference between the direct and reflected signals delays, $P_s(2\pi f)$ is the signal power on the frequency f:

$$P_s(\omega) = \frac{K}{N} P \left| \sum_{k \in mask} G_{f,k}(\omega - \omega_0 - \omega_k) \right|^2 \tag{10}$$

where $G_{f,k}(\omega)$ is the Fourier transform for modulation function $G_k(t)$ on the k-th subcarrier.

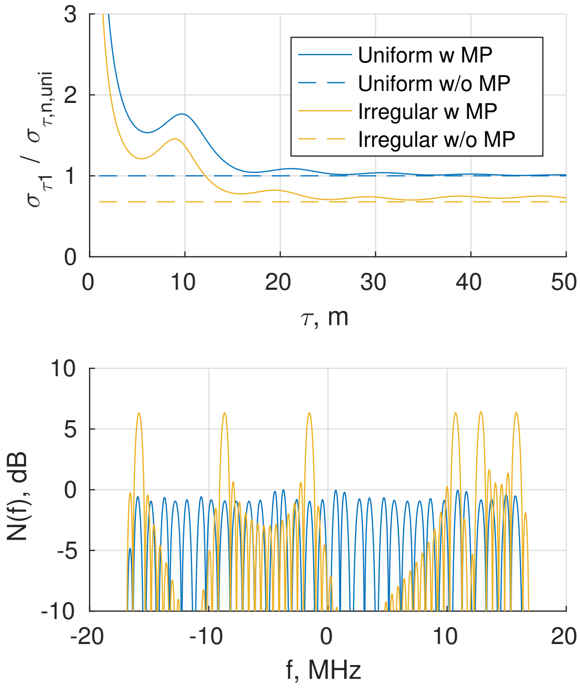

Figure 5: Signal spectrum for the proposed modulation scheme

CRB for delay estimation root-mean-square error is:

$$\sigma_{\tau 1} = \sqrt{\left(\mathbf{J}_{\theta}^{-1}\right)_{1,1}} \tag{11}$$

The errors (11) graphs are depicted on the figure 5 for OFDM signals with uniform and irregular subcarriers (normalized to potential delay error $\sigma_{\tau,n,uni}$). The subcarriers are spaced by $\delta f = 1.023$ MHz, $N = 32$, $K = 6$ and modulated by BPSK with the 1.023 MHz rate. The channel attenuations are $\alpha_1 = 1$, $\alpha_2 = 0.5$.

As follows from the graphs, the signal with a irregular distribution of subcarriers has the smaller delay estimation error even in the multipath conditions. The RMSE gain is about 20-30 percents in the presented case.

3.2. Interference immunity

It is proposed to use different subcarrier masks for signals of different beacons (fig. 6). In this case signal components with internal subcarriers use different frequencies. It allows to increase an anti-jam capability and decrease an intrasystem interference.

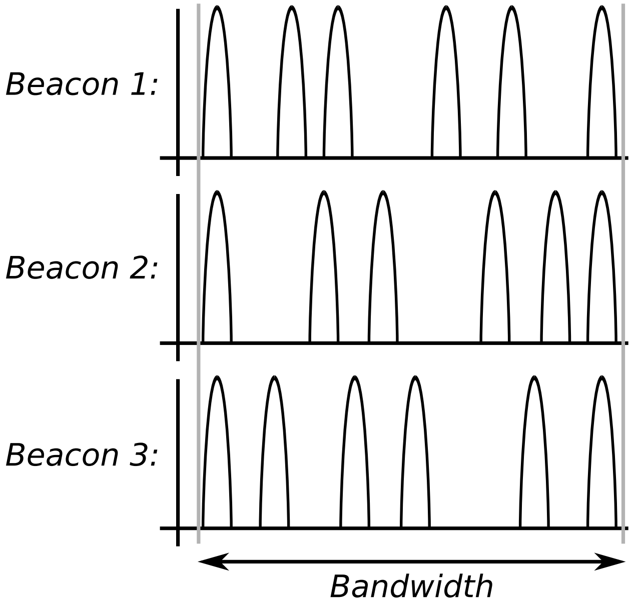

Figure 6: Individual subcarriers for different beacons

In same time, subcarriers on the edges are common. It allows to use signal processing algorithms which depends on a group delay for various beacons (like a phase ambiguity resolution). The gain of anti-jam capability can be written (relatively to the uniform subcarriers case):

$$\frac{K_{irreg}}{K_{uni}} = \frac{\left(2 + (N-2)M\right)\frac{K}{N}}{K} = \frac{2 + (N-2)M}{N}, \tag{12}$$

where M is the number of beacons.

If the number is sufficient large, then the gain in anti-jam capability is proportional to the beacons number. For example, for GNSS the gain is about 10-15 dB.

3.3. Acquisition sensitivity

The fact that N is relatively small is an advantage for the signal **acquisition** procedure. Each doubling of the number of subcarriers (all other things being equal) causes a loss of 1 dB of the acquisition procedure sensitivity in the condition of limited computing resources [6]. The utilization irregular subcarriers instead of uniform ones allows to increase the acquisition sensitivity to:

$$Se_{irreg} - Se_{uni} \approx log_2\left(\frac{K}{N}\right)[dB]\,. \tag{13}$$

For $K = 6$, $N = 32$ we have got the 2.5 dB gain.

4. CONCLUSION

The orthogonal frequency multiplexing as a modulation technique for measuring signals of navigation and location systems is discussed. It is shown that the technique has several advantages in comparison on traditional BOC and BPSK. The OFDM allows to form complex spectrum shapes and to process signal components separately.

It is shown the uniform subcarrier distribution of traditional OFDM signal doesn't allow to achieve maximum ranging accuracy and interference immunity. Similarly, using of only last subcarriers is not optimal in multipath conditions.

An OFDM with irregular subcarriers approach is proposed.

Relatively to OFDM with uniform subcarriers the approach allows to increase the delay estimation accuracy by tens of percent even in multipath conditions. An anti-jam capability is increased proportionally on the number of beacons (about 10-15 dB gain for GNSS).

It is possible to generate the signal with the constant envelope to increase the transmitter efficiency. Reducing the number of subcarriers leads to increasing of acquisition sensitivity.

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